Squeezed light in integrated photonic structures

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Representing the time evolution operator of a multimode quadratic Hamiltonian as an operator product of $S(z)$, $D(\alpha)$, and $R(\Phi)$.

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Characterising the photon statistics generated via an effective $\chi^{(2)}$ interaction process in a ring resonator system.

Theory: Asymptotic in/out states and effective $\chi^{(2)}$ interaction in an integrated photonic structure

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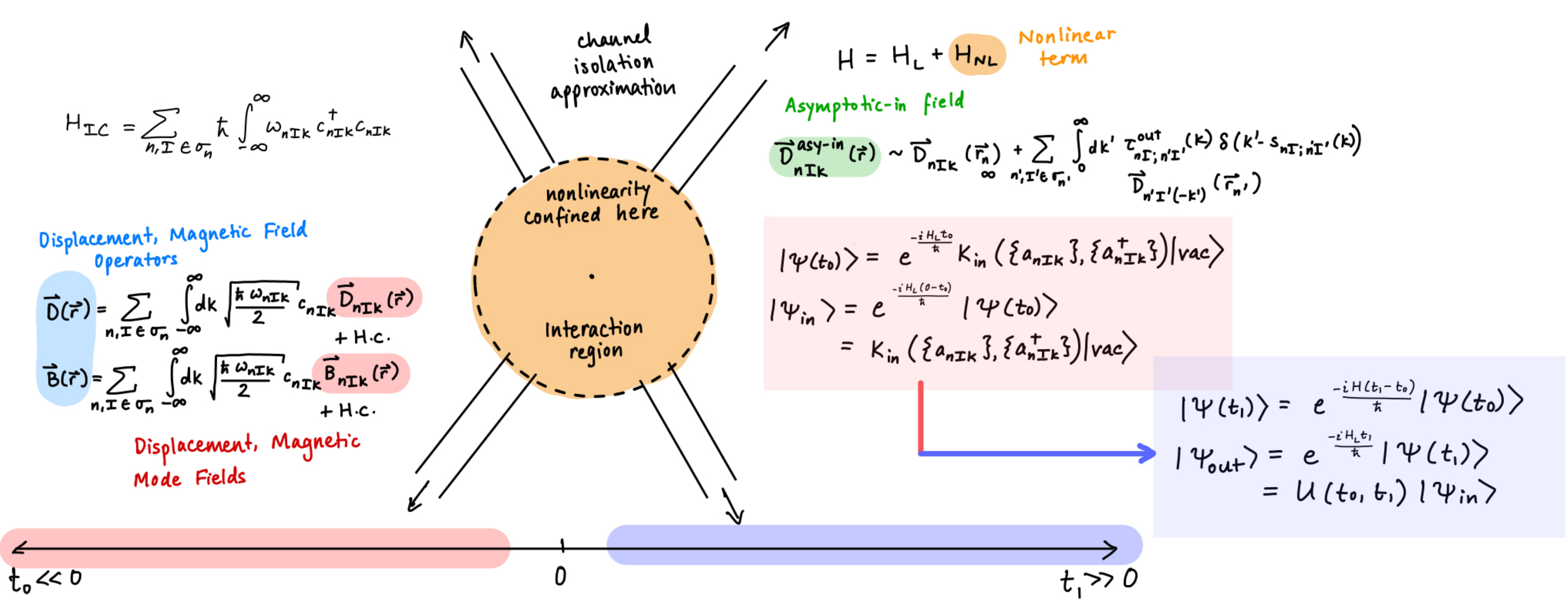

Outline of the general asymptotic in/out ket formulation of the photon generation process, with nonlinearity spatially restricted.

Liscidini, M., L. G. Helt, and J. E. Sipe. â€œAsymptotic Fields for a Hamiltonian Treatment of Nonlinear Electromagnetic Phenomena. *Physical Review A* 85, no. 1 (January 23, 2012). https://doi.org/10.1103/physreva.85.013833.

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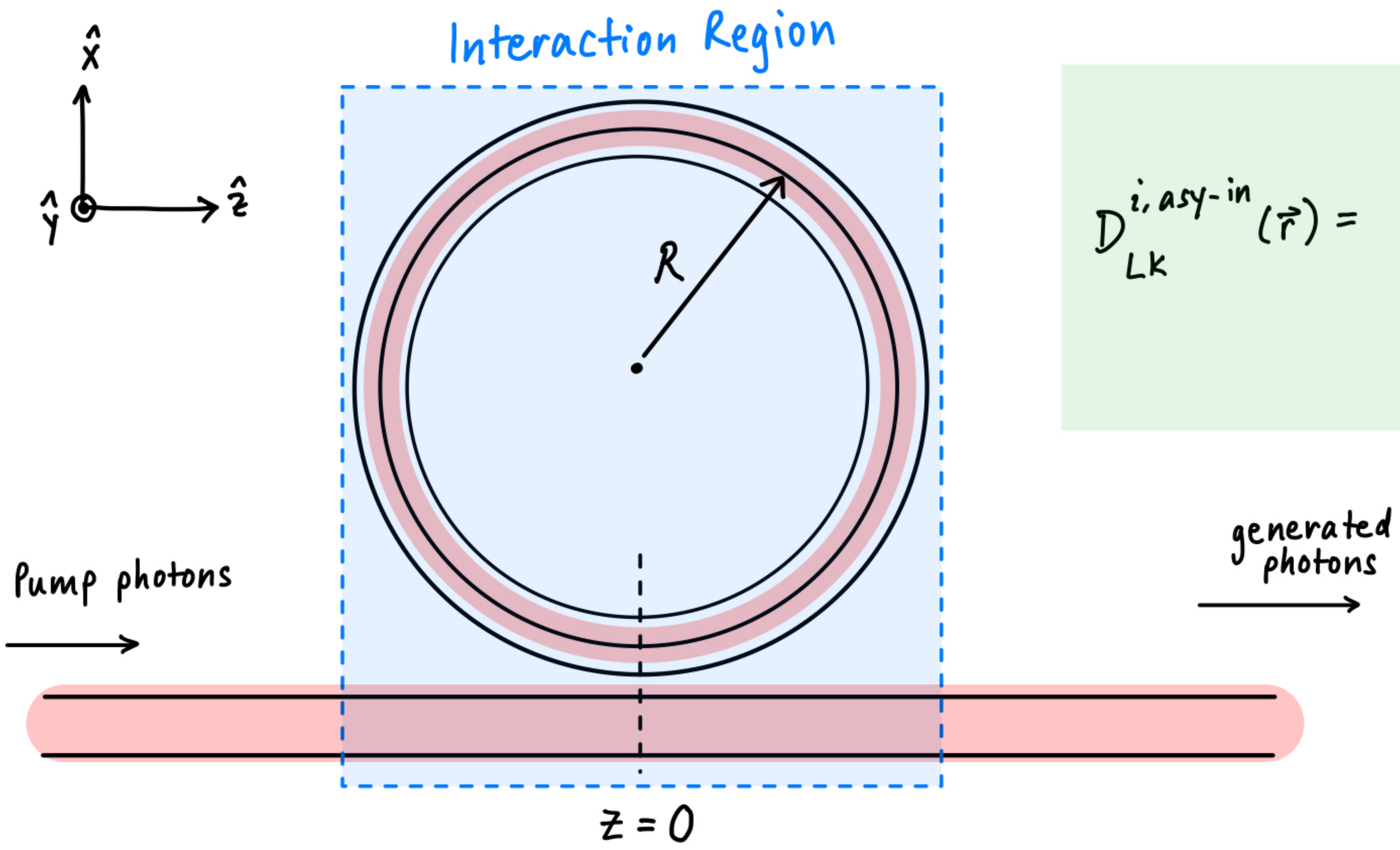

$$D^{i,asy\text{-}in}_{Lk}(\vec{r}) = \begin{cases} \frac{1}{\sqrt{2\pi}}\, d^{i}_{k}(\vec{r}_\perp)\, e^{ikz}, & z < 0 \\ \frac{T(k)}{\sqrt{2\pi}}\, d^{i}_{k}(\vec{r}_\perp)\, e^{ikz}, & z > 0 \\ + D^{i,ring}_{Lk}(\vec{r}) & \text{in ring} \end{cases}$$

Application: Spontaneous parametric down conversion (SPDC) in a microring resonator

Theory: Quadratic Hamiltonians and their unitary evolution operators

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Ma, Xin, and William Rhodes. Multimode Squeeze Operators and Squeezed States. Physical Review A 41, no. 9 (May 1, 1990): 4625-31. https://doi.org/10.1103/physreva.41.4625.

Hamiltonian

$$\hat{H}_N(t) = (\hat{a}^\dagger)^\intercal \omega(t)\hat{a} + (\hat{a}^\dagger)^\intercal f(t)\hat{a}^\dagger + (\hat{a})^\intercal f^\dagger(t)\hat{a} + g^\intercal(t)\hat{a}^\dagger + g^\dagger(t)\hat{a} + h(t)$$

Time evolution

$$\hat{U}_N(t) = \exp\left[i\gamma_N(t)\right]\hat{S}_N(z)\hat{D}_N(\alpha)\hat{R}_N(\Phi)$$

Squeezing, rotation, and displacement operator definitions

$$\hat{S}(z) \equiv \exp\left(\frac{z(\hat{a}^\dagger)^2}{2} - \frac{z^*\hat{a}^2}{2}\right)$$
$$\hat{D}(\alpha) \equiv \exp\left(\alpha\hat{a}^\dagger - \alpha^*\hat{a}\right)$$
$$\hat{R}(\phi) \equiv \exp\left(i\phi\hat{a}^\dagger\hat{a}\right)$$

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Disentangling the operator product:

Disentangling individual operators

$\downarrow$

Normal ordering via commutation relations

$$\hat{S}_N(z)\hat{D}_N(\alpha)\hat{R}_N(\Phi) = |S|^{\frac{1}{2}} \exp\left[-\frac{1}{2}(\alpha^\dagger\alpha + \alpha^\intercal T^\dagger \alpha)\right] \exp\left[\alpha^\intercal S^\intercal \hat{a}^\dagger + \frac{1}{2}(\hat{a}^\dagger)^\intercal T \hat{a}^\dagger\right]$$
$$\times \left[\sum_{n=0}^{\infty} \frac{:[(\hat{a}^\dagger)^\intercal (S e^{i\Phi} - I)\hat{a}]^n:}{n!}\right] \exp\left[-(\alpha^\intercal T^\dagger + \alpha^\dagger) e^{i\Phi}\hat{a} - \frac{1}{2}\hat{a}^\intercal e^{i\Phi^\intercal}\hat{a}\right]$$

The final normal ordered form

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$$\hat{U}_N(t) = \exp\left[A(t)\right] \exp\left[B^\intercal(t)\hat{a}^\dagger + (\hat{a}^\dagger)^\intercal C(t)\hat{a}^\dagger\right] \left[\sum_{n=0}^{\infty} \frac{:[(\hat{a}^\dagger)^\intercal D(t)\hat{a}]^n:}{n!}\right] \exp\left[E^\intercal(t)\hat{a} + \hat{a}^\intercal F(t)\hat{a}\right]$$

Schrödinger

$$\begin{aligned}
i\frac{\partial}{\partial t}A(t) &= \mathrm{Tr}\left[f^\dagger(2C(t) + B(t)B^\intercal(t)) + g^\dagger(t)B(t) + h(t)\right], \\
i\frac{\partial}{\partial t}B(t) &= (4C(t)f^\dagger(t) + \omega(t))B(t) + 2C(t)g^*(t) + g(t), \\
i\frac{\partial}{\partial t}C(t) &= 4C(t)f^\dagger(t)C(t) + 2\omega(t)C(t) + f(t), \\
i\frac{\partial}{\partial t}D(t) &= (4C(t)f^\dagger(t) + \omega(t))(D(t) + I), \\
i\frac{\partial}{\partial t}E(t) &= (D^\intercal(t) + I)(2f^\dagger(t)B(t) + g^*(t)), \\
i\frac{\partial}{\partial t}f(t) &= (D^\intercal(t) + I)f^\dagger(t)(D(t) + I).
\end{aligned}$$

Theory: Extracting the squeezing and rotation matrices' elements

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Schrödinger Equation

$$\begin{cases} i\hbar\frac{dU(t)}{dt} = H(t)U(t) \\ U(t_0) = 1, \quad H(t) = \hbar\sum_{k,l}\Delta_{kl}(t)a_k^\dagger a_l + \hbar\sum_{k,l}\zeta_{kl}(t)a_k^\dagger a_l^\dagger + H.c. \end{cases} \implies U(t) = S(t)R(t)e^{i\theta(t)}$$

Squeezing matrix polar decomposition

$$\mathbf{J} = \mathbf{u}e^{i\alpha}$$

$$\mathbf{u} = \sqrt{\mathbf{J}\mathbf{J}^\dagger}$$

Heisenberg equations for a and a†

$$\frac{d}{dt}\begin{bmatrix}\mathbf{a}\\ \mathbf{a}^\dagger\end{bmatrix} = -i\begin{bmatrix}\Delta & 2\zeta\\ -2\zeta* & -\Delta*\end{bmatrix}\begin{bmatrix}\mathbf{a}\\ \mathbf{a}^\dagger\end{bmatrix}$$

$$\begin{bmatrix}\mathbf{a}\\ \mathbf{a}^\dagger\end{bmatrix} = \begin{bmatrix}\mathbf{V} & \mathbf{W}\\ \mathbf{W}* & \mathbf{V}*\end{bmatrix}\begin{bmatrix}\mathbf{a(t_0)}\\ \mathbf{a^\dagger(t_0)}\end{bmatrix}$$

Reduction to coupled ODEs

$$\frac{d}{dt}\mathbf{V} = -i\Delta\mathbf{V} - 2i\zeta\mathbf{W}*$$

$$\frac{d}{dt}\mathbf{W} = -i\Delta\mathbf{W} - 2i\zeta\mathbf{V}*$$

ODE constraints

$$\mathbf{W}\mathbf{V}^\mathbf{T} - \mathbf{V}\mathbf{W}^\mathbf{T} = 0$$

$$\mathbf{V}\mathbf{V}^\dagger - \mathbf{W}\mathbf{W}^\dagger = 1$$

Total number of generated photons (preliminary — no phantom channel)

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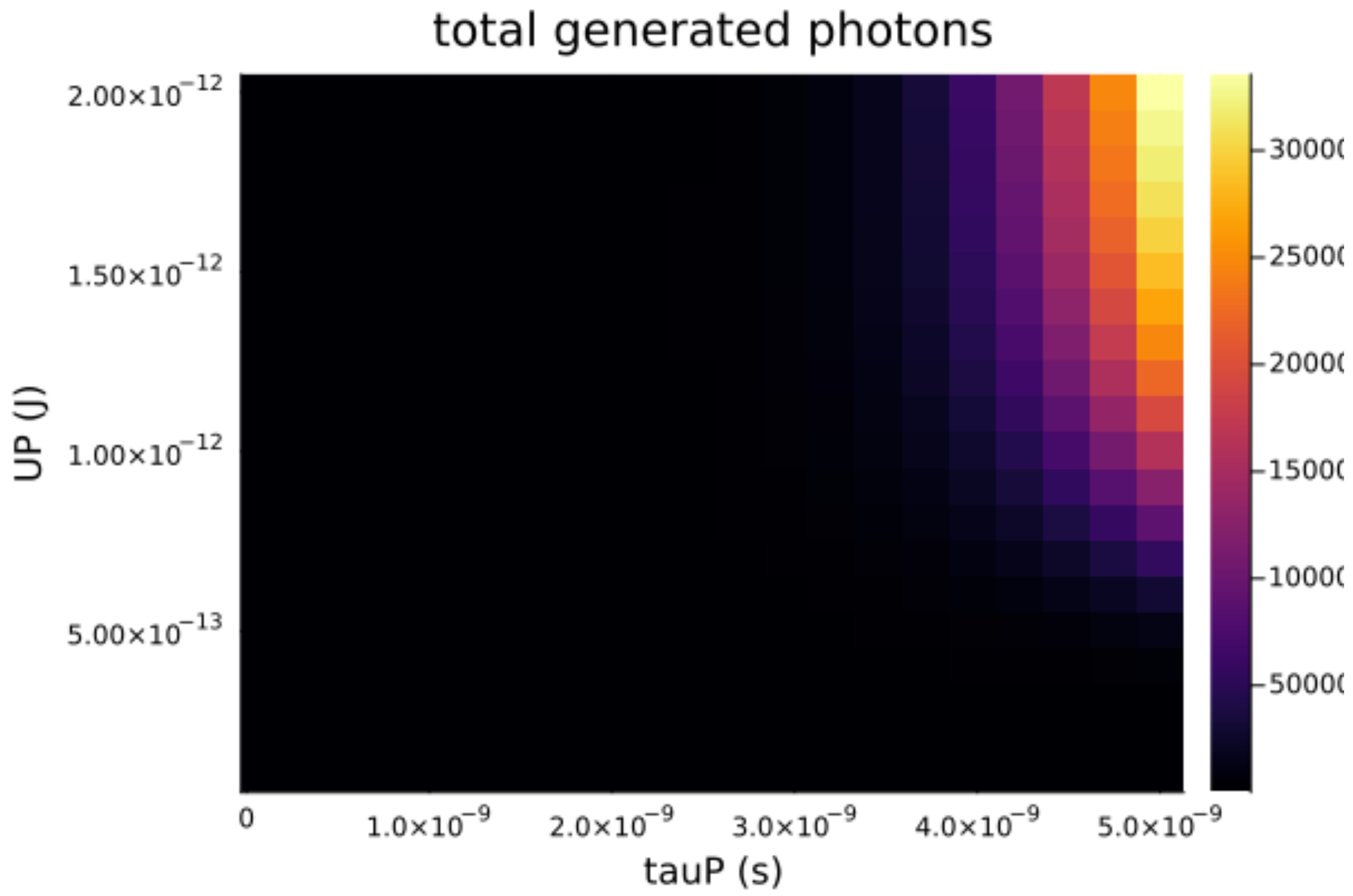
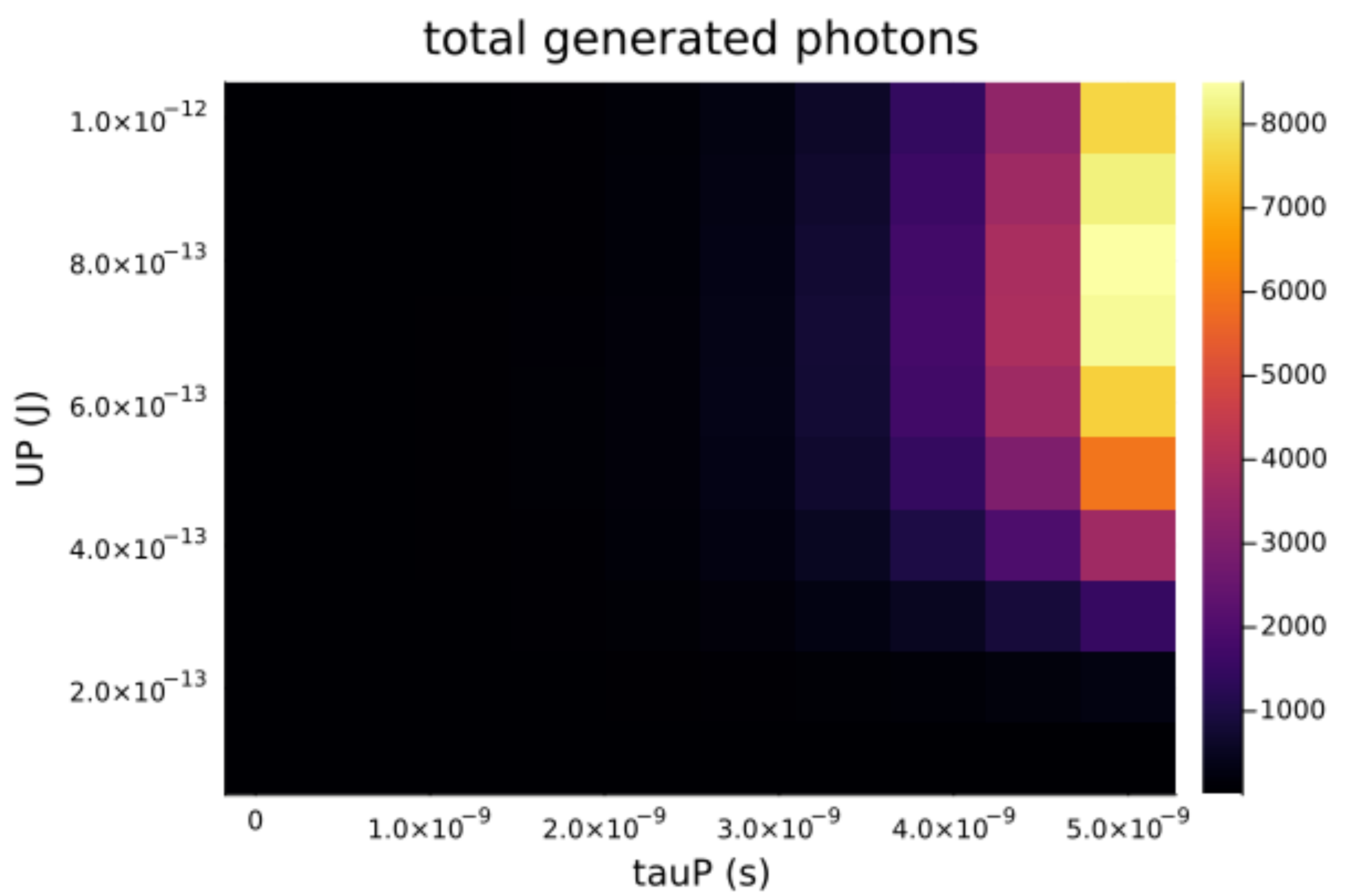
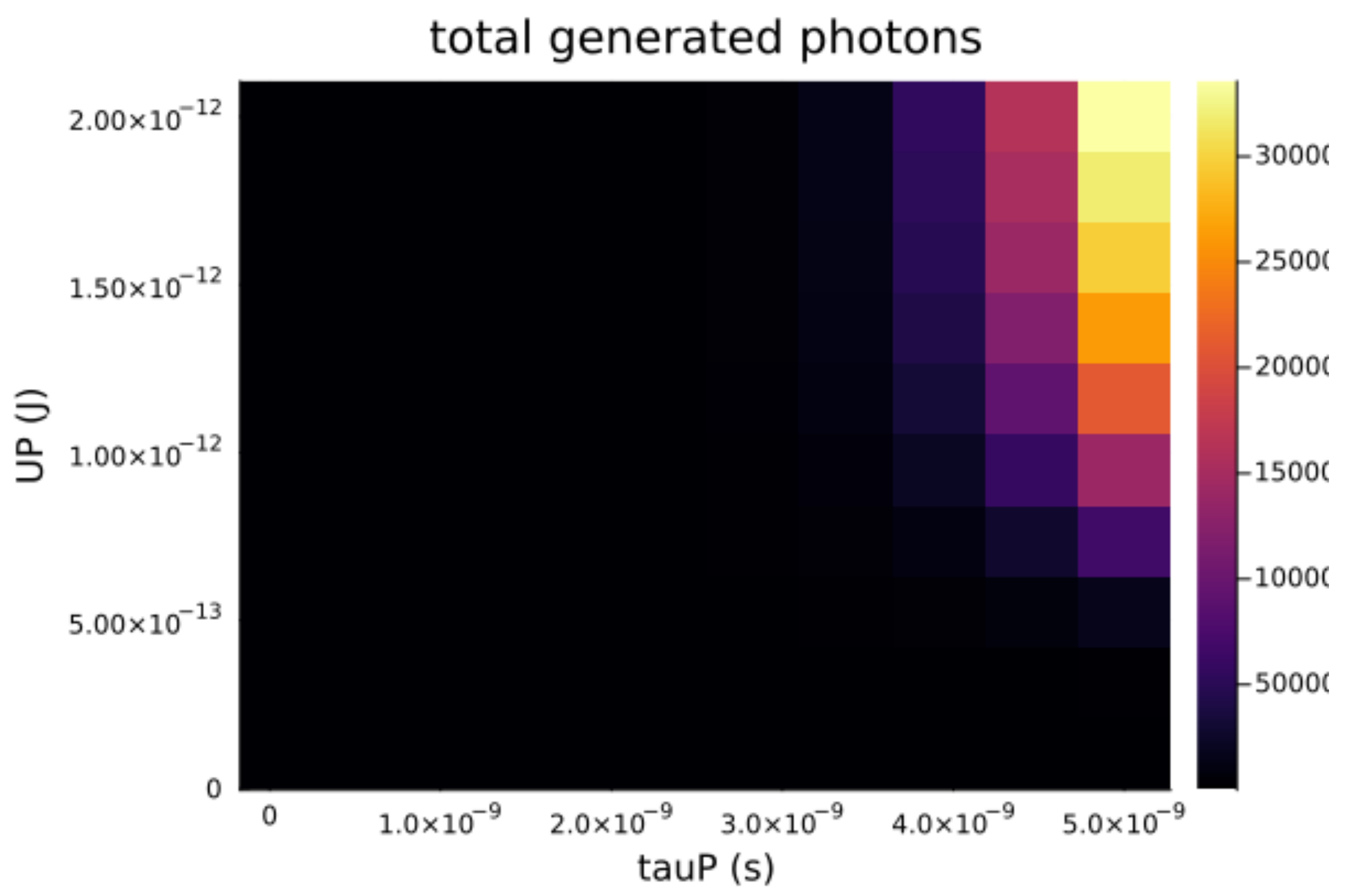


Moral: 20 x 20 resolution insufficient!



20 points in frequency space, τ = 0.1 ns - 5.0 ns & U = 0.1 pJ - 1.0 pJ
Resolution = 10 x 10



20 points in frequency space, τ = 0.1 ns - 5.0 ns & U = 0.1 pJ - 2.0 pJ
Resolution = 10 x 10

Theory: The addition of a phantom channel to model lossy generation

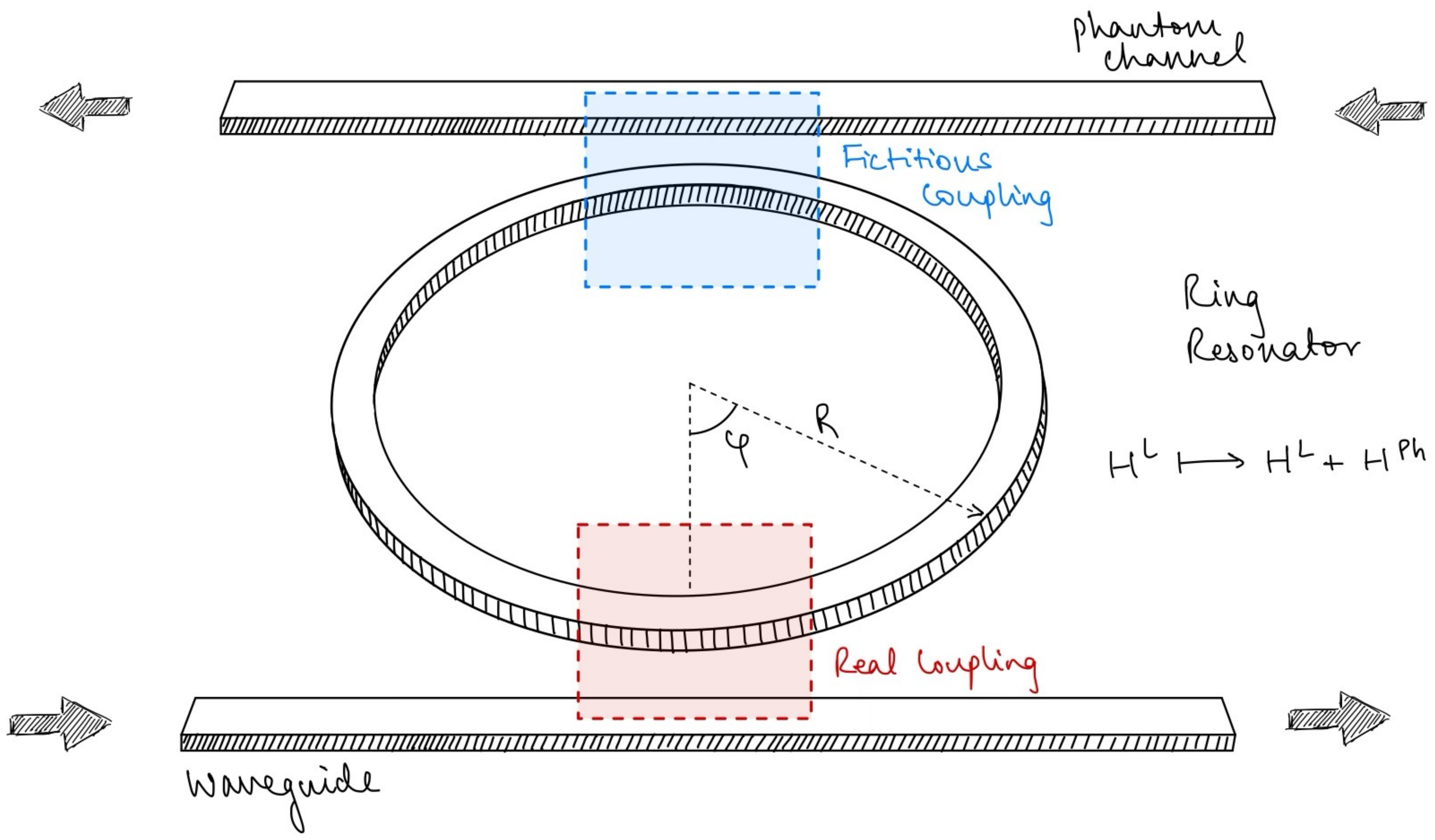
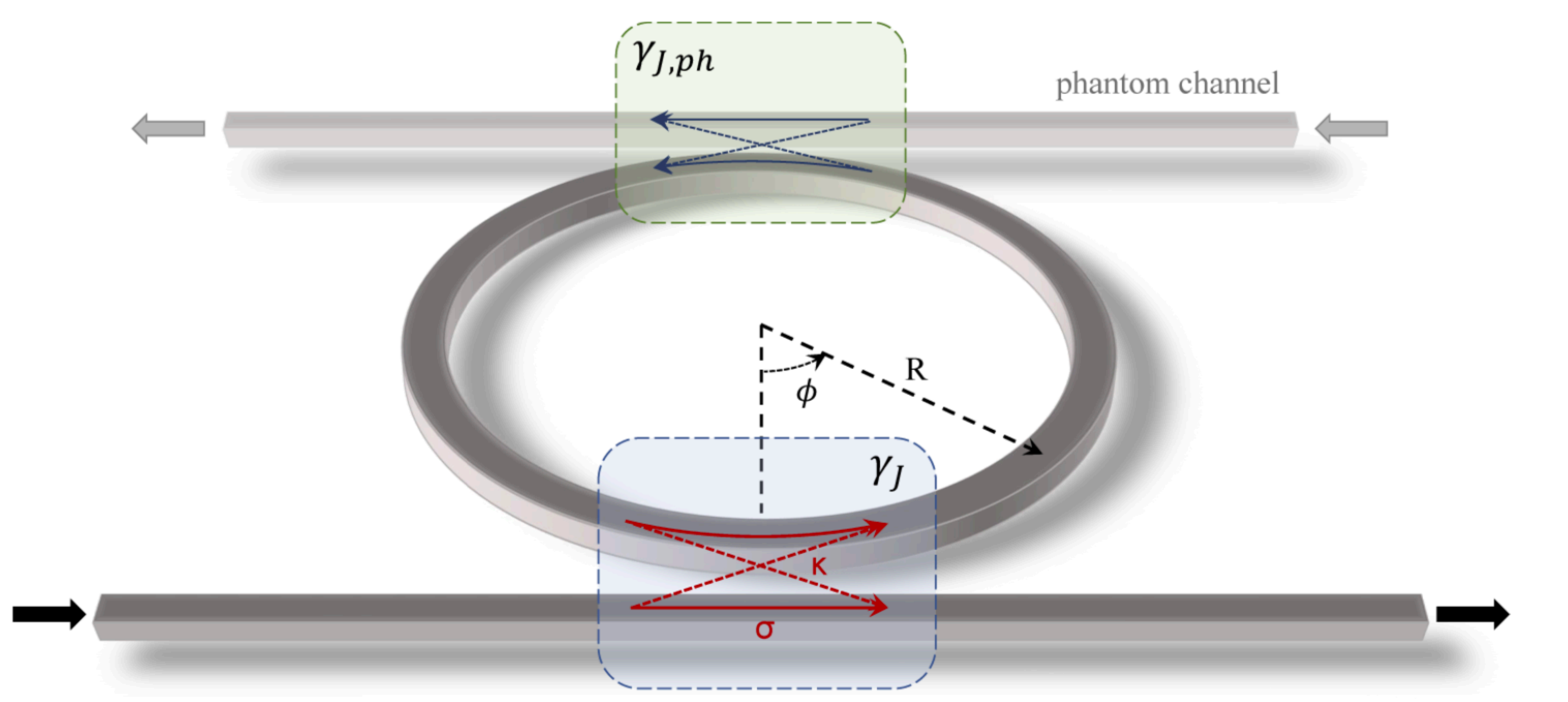


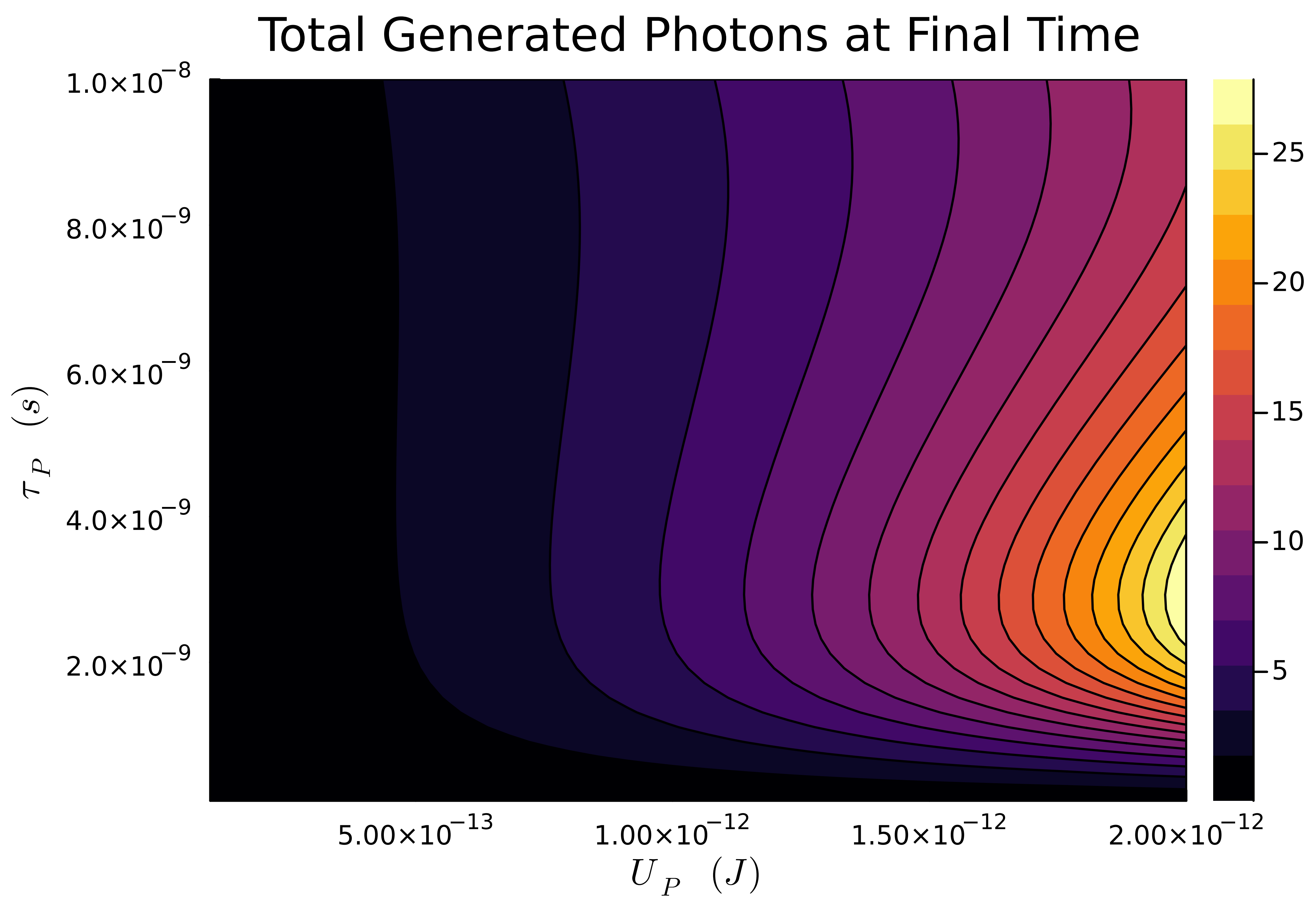
Figure 6

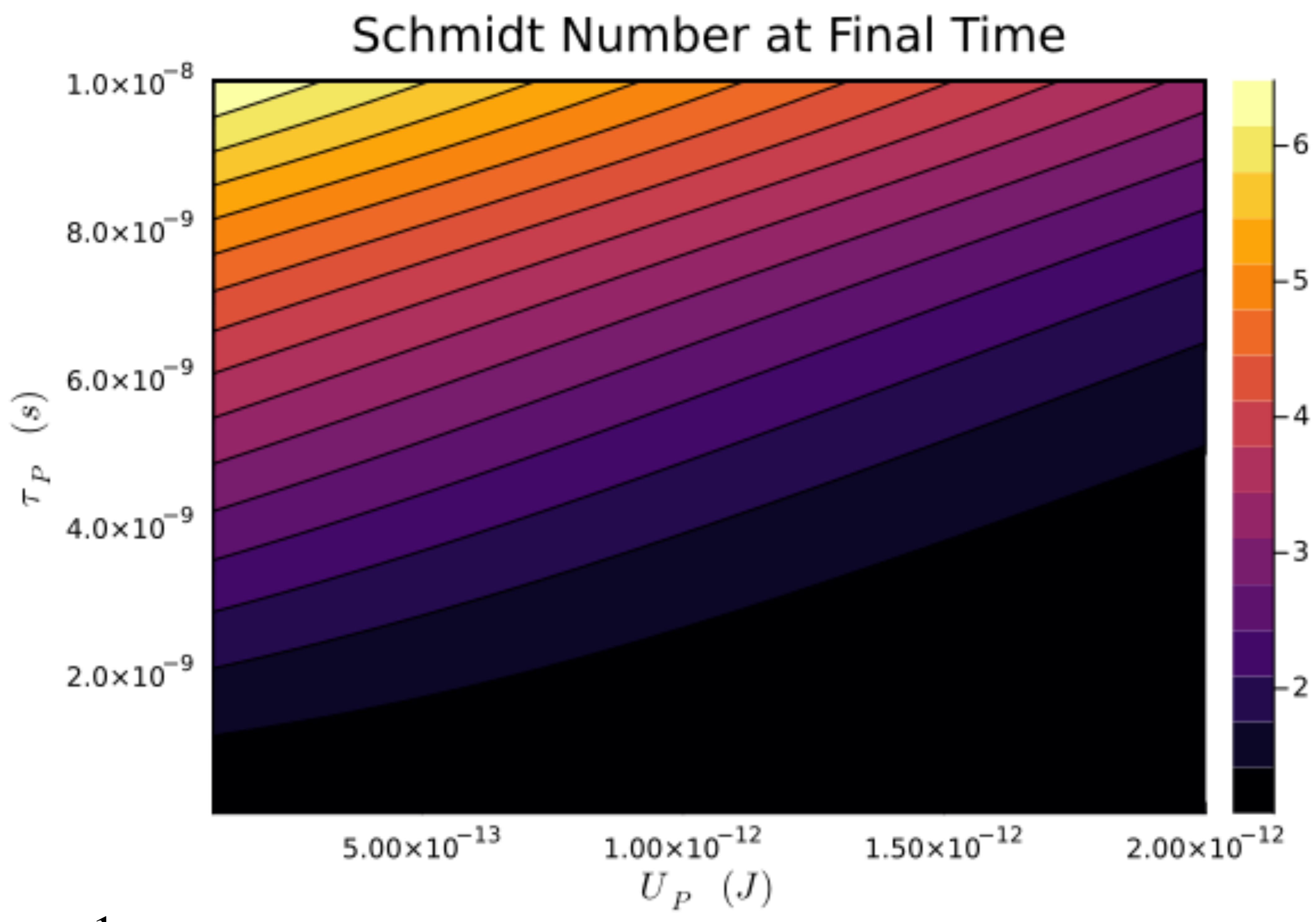


Schematic representation of a point-coupled ring resonator with and additional fictitious coupler and waveguide, referred to as “phantom channel.”

For simplicity, we make the undepleted pump approximation, i.e. the pump source is treated as unaffected by interaction. It is a valid approximation in most cases as at most a negligible fraction of the pump power is transferred to the generated fields.

Adding a phantom channel

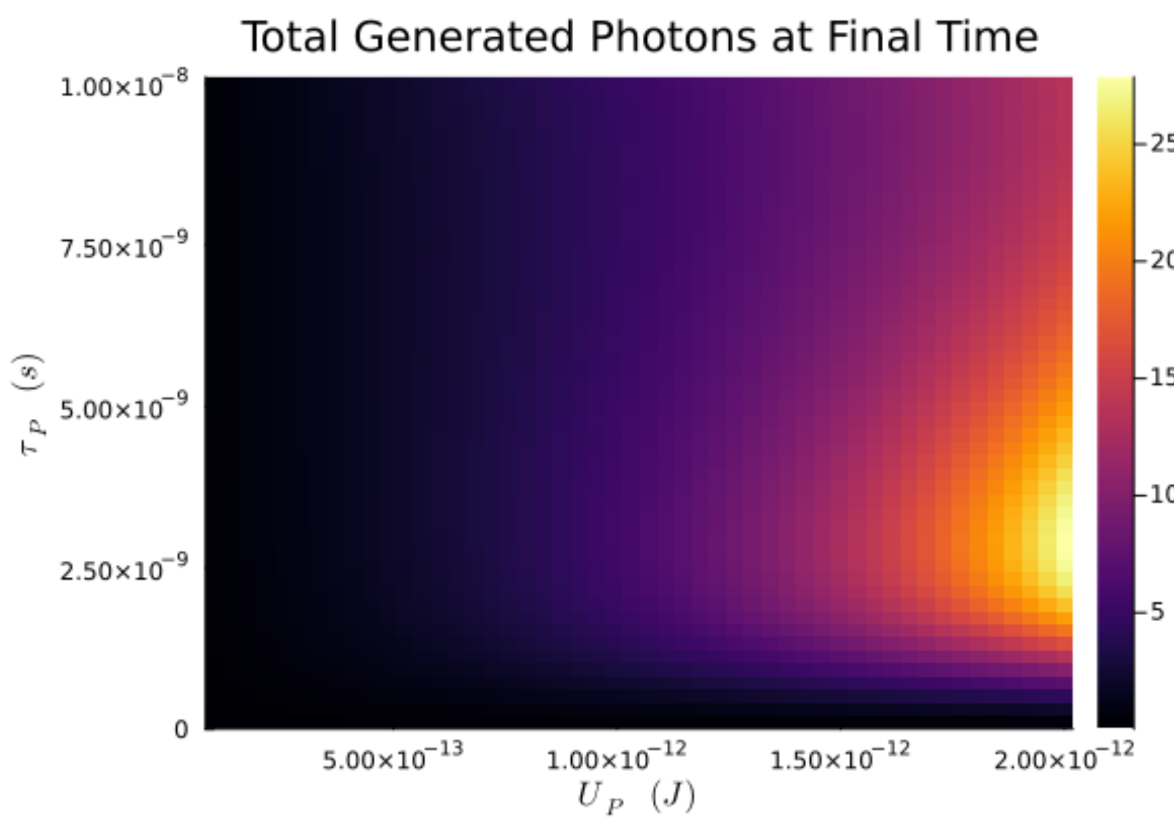


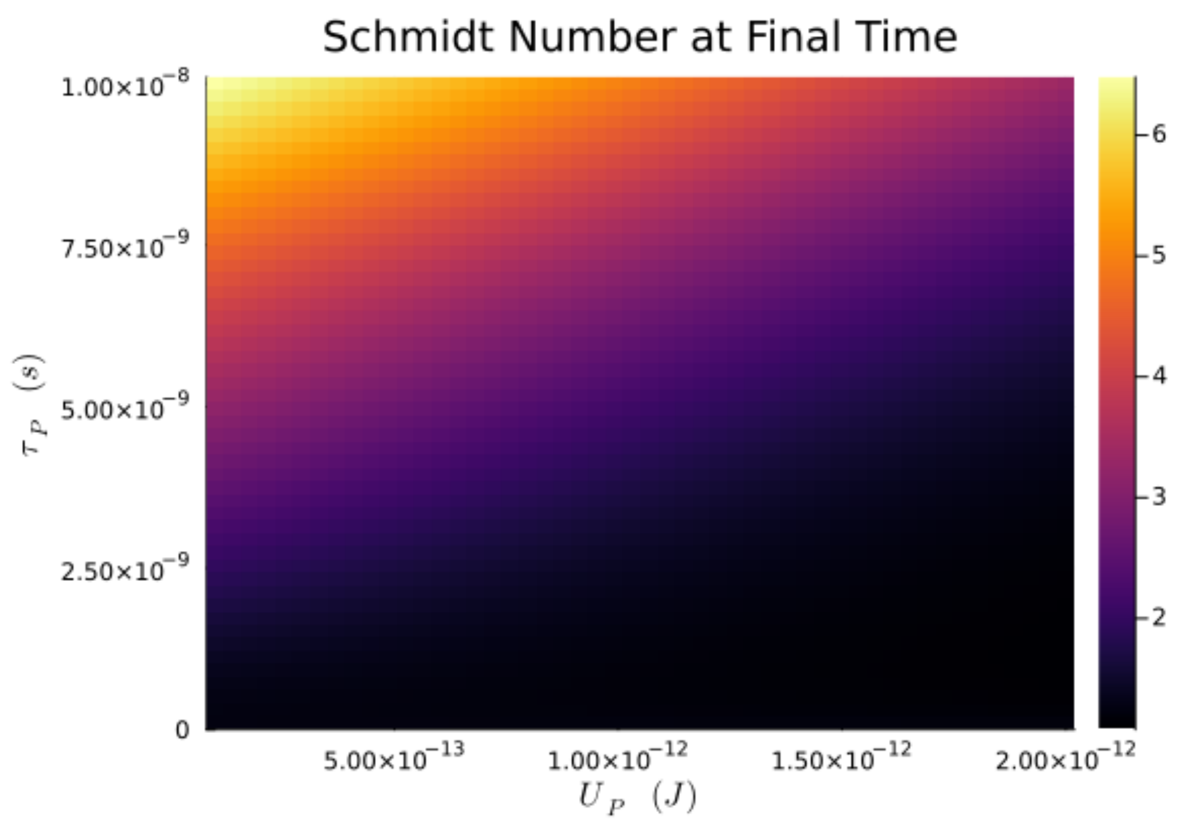


Schmidt number $K = \frac{1}{\sum_{n=0}^{\infty} \lambda_n^2}$

"Degree of entanglement"

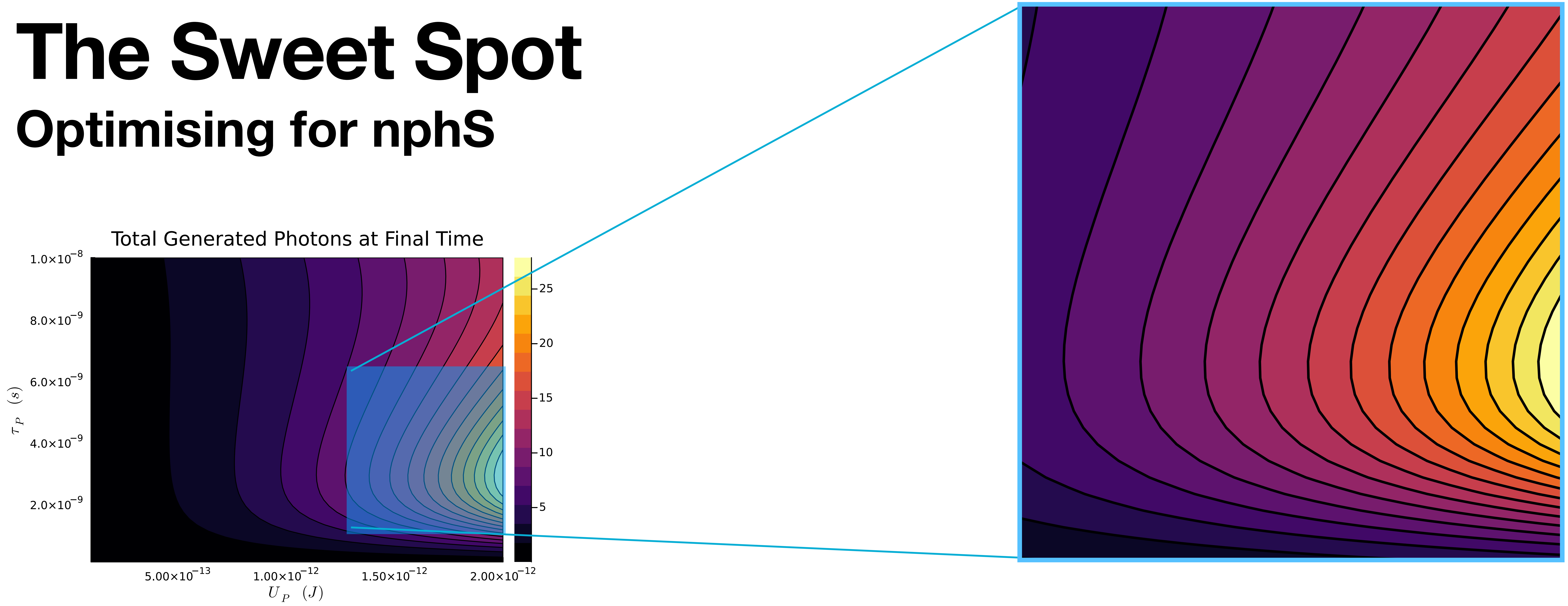
Variable k points, τ = 0.1 ns - 5.0 ns & U = 0.1 pJ - 1.0 pJ
Resolution: 50 x 50





The Sweet Spot

Optimising for nphS

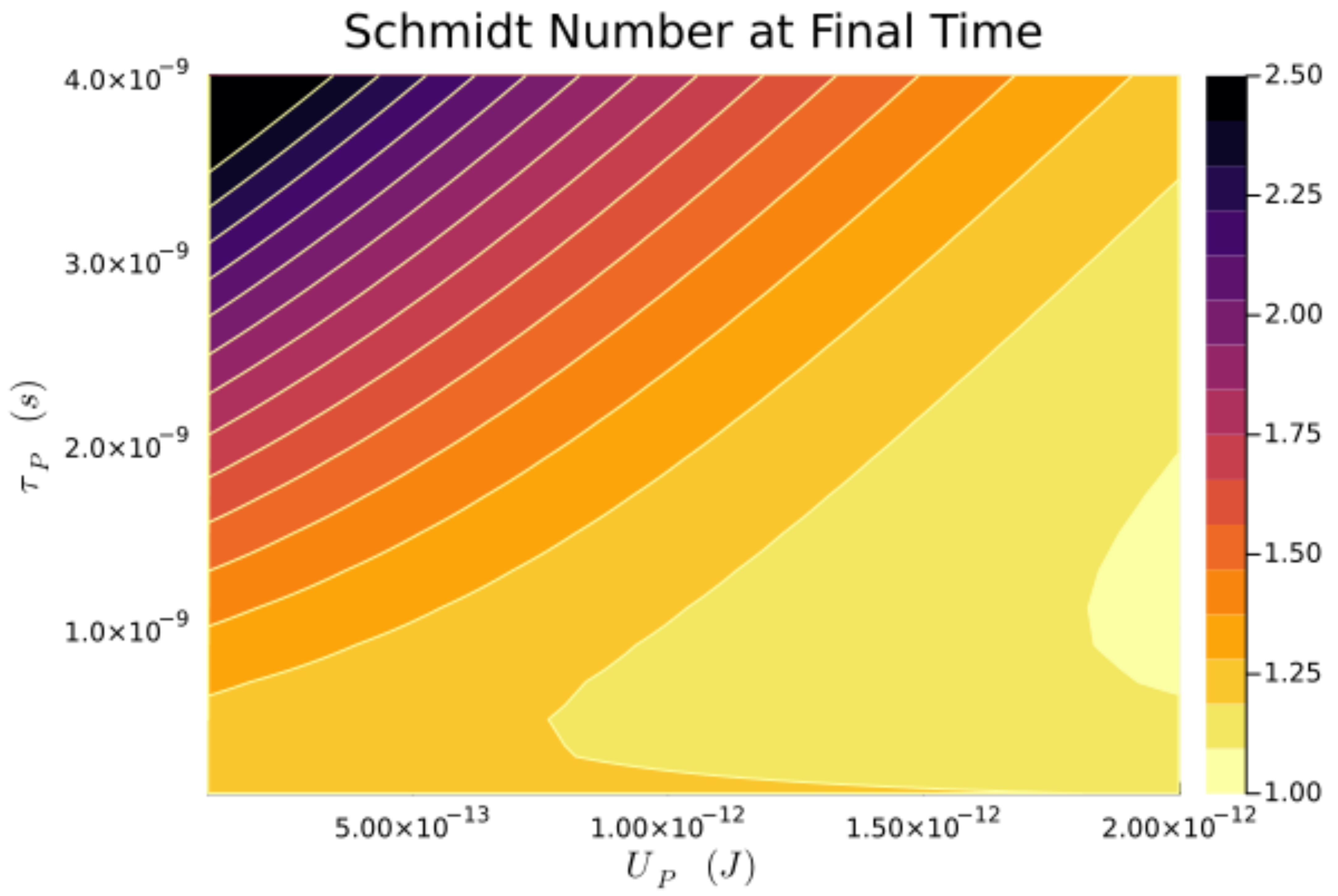


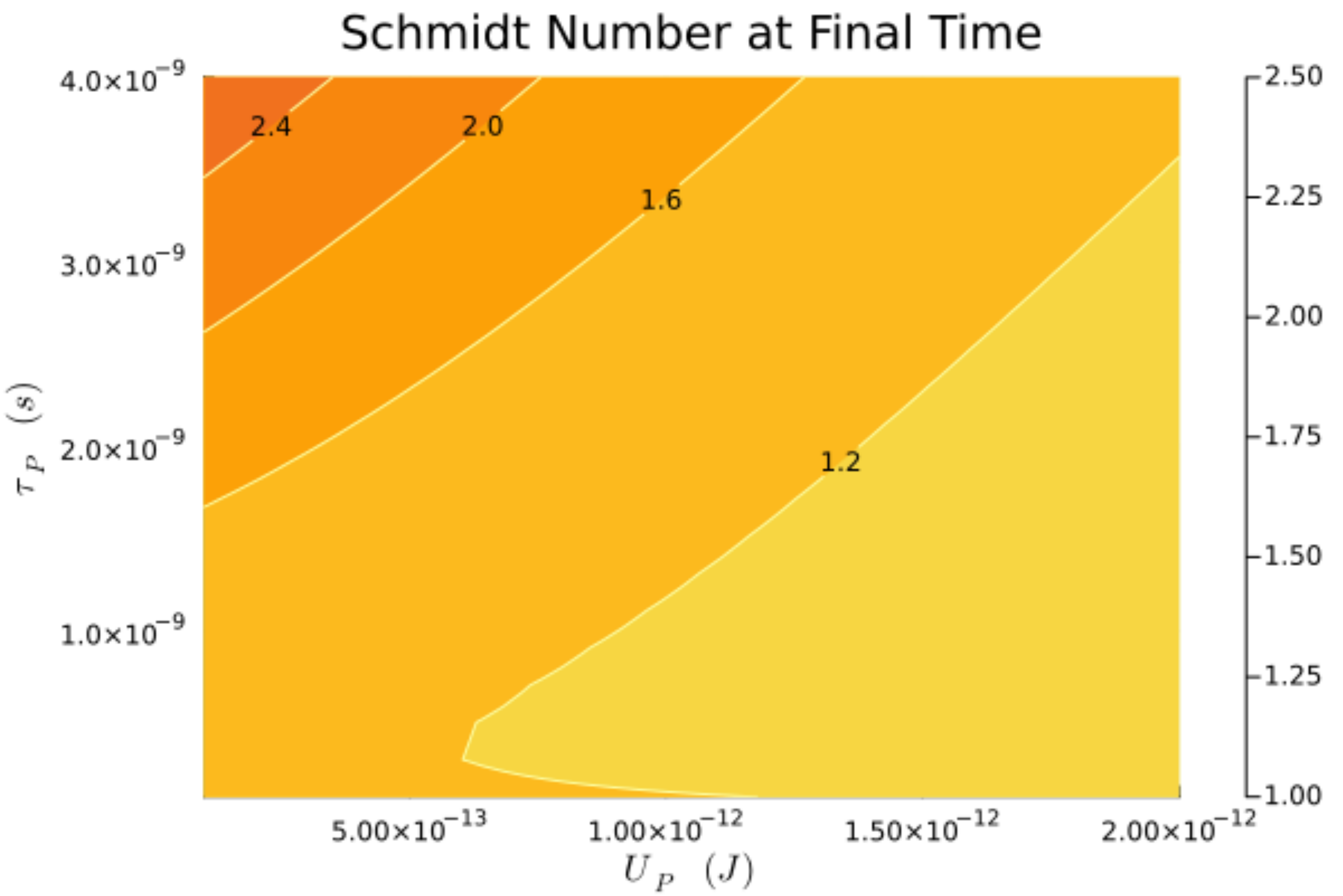
For the ranges U_P = 0.1 pJ - 2.0 pJ and τ_P = 0.1 ns - 10.0 ns, the largest total photon number generated was **~ 27.9**, for U_P **~ 2.0 pJ** and τ_P **~ 2.93 ns**.

Why? Very short τ_P means that the frequency of the pulse greatly exceeds the ring resonance width, so **only a small fraction of the pulse enters the resonator**.

Going to Schmidt Number 1.0

Returning optimal τ_P and U_P





Plotting contour lines and examining K ~ (1.1, 3.0) isoclines, we see that τ_P and U_P yielding a Schmidt number around 1.0 are around 1.0 ns and 2.0 pJ respectively.

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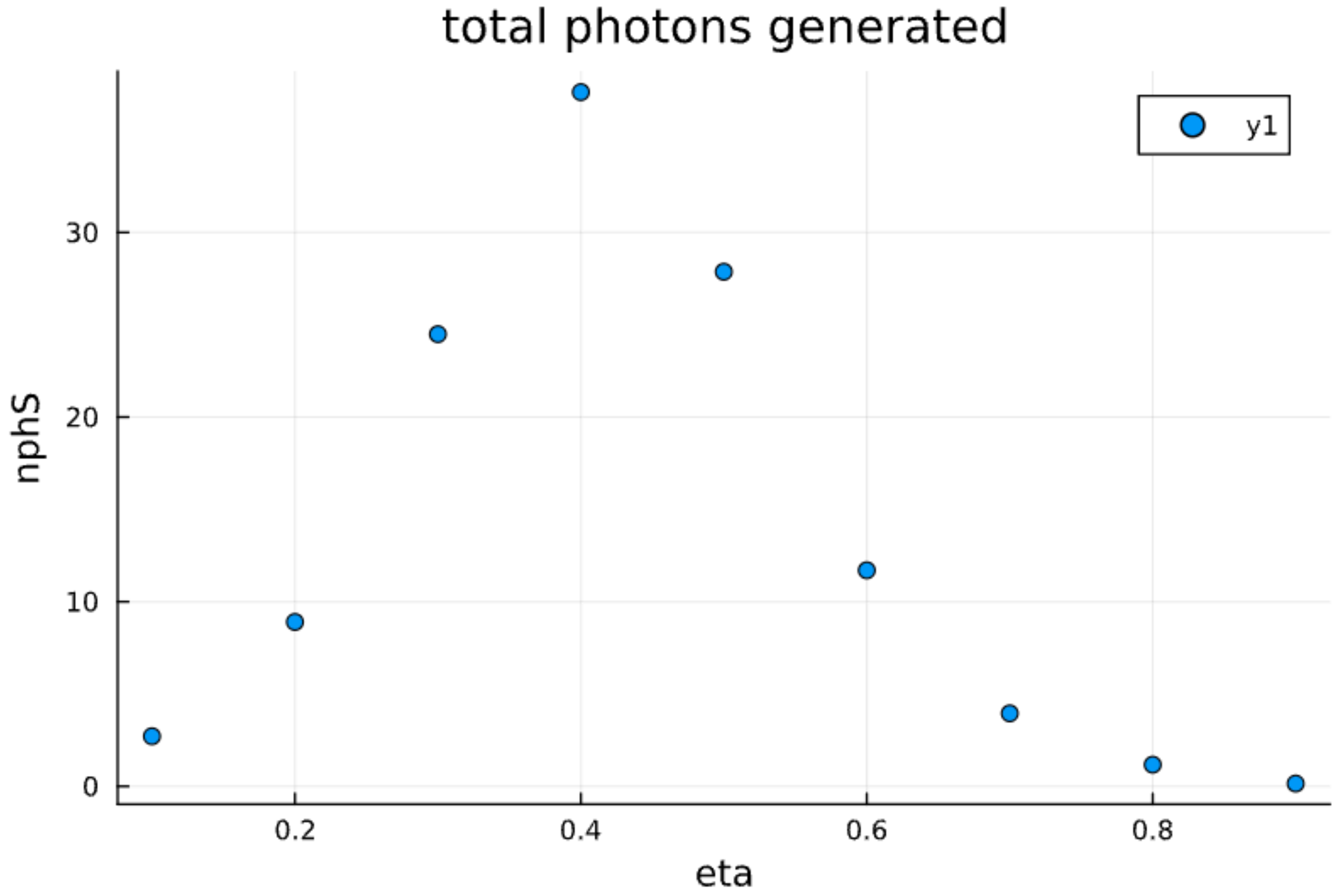
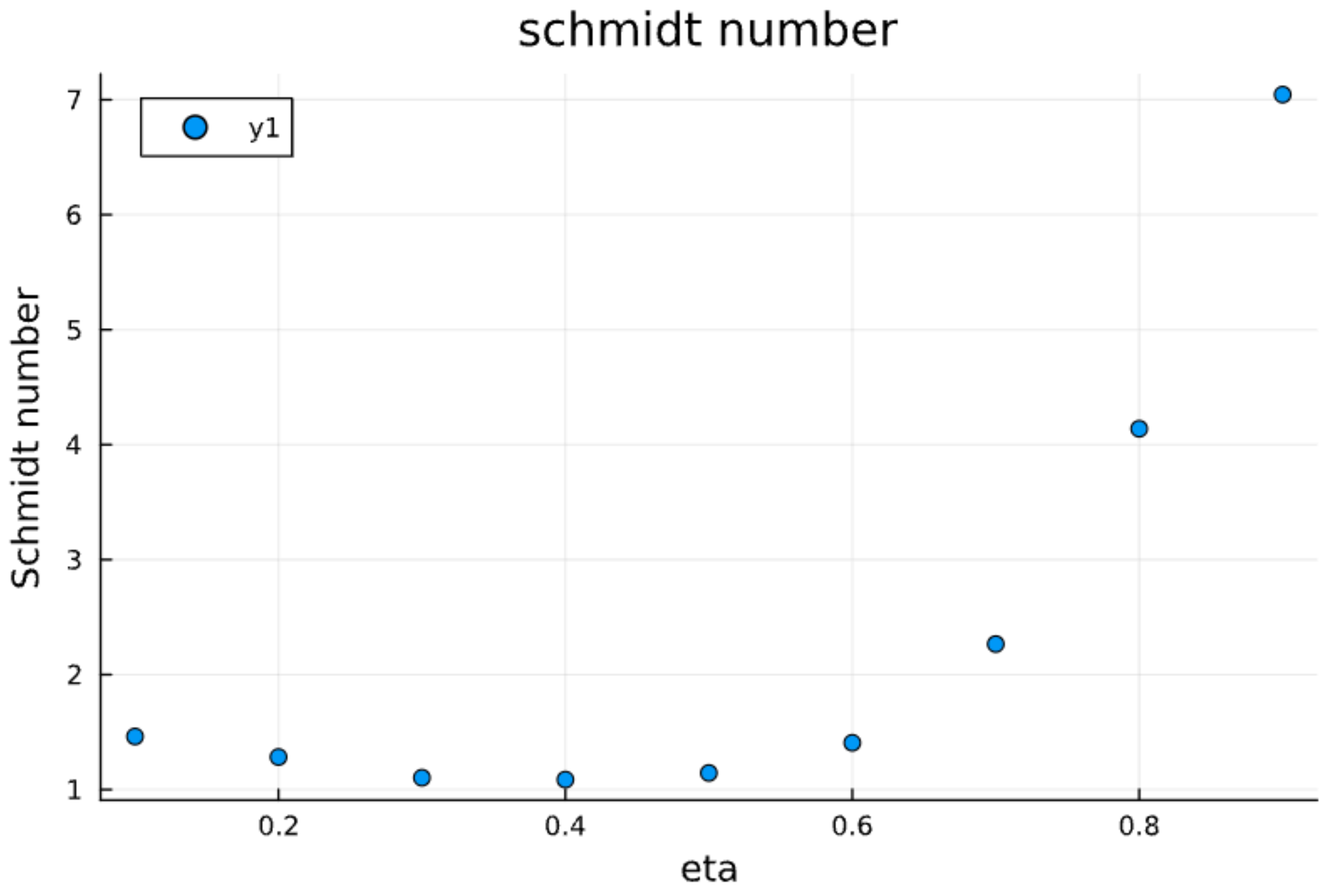
Behaviour with respect to coupling efficiency

Maximising nphS, K with respect to η

Changing $\eta = \eta_C = \eta_S = \eta_P$ from [0.1, 0.9] and observing the effects on the numbers of photon generated and the Schmidt number, we see that a peak at 0.4 in nphS corresponds to a K value closest to 1.

Agreement with Milica Banic's results.





Conclusion
Main results

Time evolution

$$\boxed{\hat{U}_N(t) = \exp\left[i\gamma_N(t)\right]\hat{S}_N(z)\hat{D}_N(\alpha)\hat{R}_N(\Phi)}$$